Chapter 1 Real Number

Question-1

Write the following rational numbers in decimal form:

(i) $\frac{42}{100}$

(ii) $\frac{327}{500}$

(iii) $3\frac{3}{8}$

(iv) $\frac{5}{6}$

(v) $\frac{1}{5}$

(vi) $\frac{1}{7}$

(vii) $\frac{2}{13}$

(viii) $\frac{11}{17}$

Solution:

(i) $\frac{42}{100} = 0.42$

(ii) $\frac{327}{500} = 0.654$

```
      0. 654
5 | 3.27
    30
    27
    25
      20
      20
       0
```

(iii) $3\frac{3}{8} = \frac{27}{8} = 3.375$

```
      3.375
8 | 27
    24
     30
     24
      60
      56
       40
       40
        0
```

(iv) $\frac{5}{6} = 0.833... = 0.8\overline{333}$

$0.8\overline{333}$
6 | 50
30
20
18
20
18
20
18
2

(v) $\frac{1}{5} = 0.2$

0.2
5 | 10
10
0

(vi) $\frac{1}{7} = 0.\overline{142857}$

$0.\overline{142857}$
7 | 10
7
30
28
20
14
60
56
40
35
50
49
1

(vii) $\frac{2}{13} = 0.\overline{153846}$

$0.\overline{153846}$
13 | 20
13
70
65
50
39
110
104
60
52
80
78
2

(viii) $\frac{11}{17} = 0.\overline{6470588235294117}$

```
      0.6470588235294117
17 )  110
      102
       80
       68
      120
      119
        100
         85
         150
         136
          140
          136
            40
            34
             60
             51
              90
              85
               50
               34
               160
               153
                 70
                 68
                  20
                  17
                   30
                   17
                   130
                   119
                    11
```

Question-2

If a is a positive rational number and n is a positive integer greater than 1, prove that a^n is a rational number.

Solution:

We know that product of two rational number is always a rational number.

Hence if a is a rational number then

$a^2 = a \times a$ is a rational number,

$a^3 = a^2 \times a$ is a rational number,

$a^4 = a^3 \times a$ is a rational number,

...

...

$\therefore a^n = a^{n-1} \times a$ is a rational number.

Question-3

Find three rational numbers lying between 0 and 0.1. Find twenty rational numbers between 0 and 0.1. Give a method to determine any number of rational numbers between 0 and 0.1.

Solution:

The three rational numbers lying between 0 and 0.1 are 0.01, 0.05, 0.09.

The twenty rational numbers between 0 and 0.1 are 0.001, 0.002, 0.003, 0.004, … 0.011, 0.012, … 0.099.

To determine any number of rational numbers between 0 and 0.1 insert 0 after the decimal.

Question-4

Complete the following:

(i) Every point on the number line corresponds to a ______________ number which may be either ______________ or ______________.

(ii) The decimal form of an irrational number is neither ______________ or ______________.

(iii) The decimal representation of the rational number $\frac{8}{27}$ is ______________.

(iv) 0 is ______________ number. [Hint: a rational /an irrational]

Solution:

(i) Every point on the number line corresponds to a real number which may be either rational or irrational.

(ii) The decimal form of an irrational number is neither recurring or terminating.

(iii) The decimal representation of the rational number $\frac{8}{27}$ is $0.\overline{296}$

(iv) 0 is a rational number.

Question-5

Which of the following rational numbers have the terminating decimal representation?

(i) 3/5 (ii)7/20 (iii)2/13

(iv) 27/40 (v) 133/125 (vi) 23/7

[Making use of the result that a rational number p/q where p and q have no common factor(s) will have a terminating representation if and only if the prime factors of q are 2's or 5's or both.]

Solution:

(i) The prime factor of 5 is 5. Hence 3/5 has a terminating decimal representation.

(ii) $20 = 4 \times 5 = 2^2 \times 5$.

The prime factors of 20 are both 2's and 5's. Hence 7/20 has a terminating decimal.

(iii) The prime factor of 13 is 13. Hence 2/13 has non-terminating decimal.

(iv) $40 = 2^3 \times 5$.

The prime factors of 40 are both 2's and 5's. Hence 27/40 has a terminating decimal.

(v) $125 = 5^3$

The prime factor of 125 is 5's. Hence 13/125 has a terminating decimal.

(vi) The prime factor of 7 is 7. Hence 23/7 has a non-terminating decimal representation.

Question-6

You have seen that $\sqrt{2}$ is not a rational number. Show that $2 + \sqrt{2}$ is not a rational number.

Solution:

Let $2 + \sqrt{2}$ be a rational number say r.

Then $2 + \sqrt{2} = r$

$\sqrt{2} = r - 2$

But, $\sqrt{2}$ is an irrational number.

Therefore, $r - 2$ is also an irrational number.

=> r is an irrational number.

Hence our assumption r is a rational number is wrong.

Hence, $2 + \sqrt{2}$ is not a rational number.

Question-7

Prove that $3\sqrt{3}$ is not a rational number.

Solution:

Let $3\sqrt{3}$ be a rational number say r.

Then $3\sqrt{3} = r$

$\sqrt{3} = (1/3)r$

(1/3) r is a rational number because product of two rational number is a rational number.

=> $\sqrt{3}$ is a rational number but $\sqrt{3}$ is not a rational number.

Therefore our assumption that $3\sqrt{3}$ is a rational number is wrong.

Question-8

Show that $\sqrt[3]{6}$ is not a rational number.

Solution:

Let $\sqrt[3]{6}$ be a rational number, say $\frac{p}{q}$ where $q \neq 0$.

Then $\sqrt[3]{6} = \frac{p}{q}$

Since $1^3 = 1$, and $2^3 = 8$, it follows that $1 < \frac{p}{q} < 2$

Then $q > 1$ because if $q = 1$ then $\frac{p}{q}$ will be an integer, and there is no integer between 1 and 2.

Now, $6 = \left(\frac{p}{q}\right)^3$

$6 = \frac{p^3}{q^3}$

$6q^2 = \frac{p^3}{q}$

q being an integer, $6q^2$ is an integer, and since $q > 1$ and q does not have a common factor with p and consequently with p^3.

So, $\frac{p^3}{q}$ is a fraction different from an integer.

Thus $6q^2 \neq \frac{p^3}{q}$.

This contradiction proves the result.

Question-9

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers.

(i) $\sqrt{4}$

(ii) $3\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{.64}$

(vi) $\sqrt{100}$

Solution:

(i) $\sqrt{4} = 2$ is rational.

(ii) $3\sqrt{18} = 3\sqrt{9 \times 2} = 3 \times 3\sqrt{2} = 9\sqrt{2}$ is irrational.

(iii) $\sqrt{1.44} = \sqrt{\frac{144}{100}} = \frac{12}{10} = 1.2$ is rational.

(iv) $\sqrt{\frac{9}{27}} = \sqrt{\frac{9}{9 \times 3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$ is irrational.

(v) $-\sqrt{0.64} = -0.8$ is rational.

(vi) $\sqrt{100} = 10$ is rational.

Question-10

In the following equations, find which of the variables x, y, z etc. represent rational numbers and which represent irrational numbers:

(i) $x^2 = 5$
(ii) $y^2 = 9$
(iii) $z^2 = 0.04$
(iv) $u^2 = 17/4$
(v) $v^2 = 3$
(vi) $w^3 = 27$
(vii) $t^2 = 0.4$

Solution:

(i) $x^2 = 5$

$\therefore x = \sqrt{5}$ is irrational.

(ii) $y^2 = 9$

$\therefore y = 3$ is rational.

(iii) $z^2 = 0.04$

$\therefore z = 0.2$ is rational.

(iv) $u^2 = \frac{17}{4}$

$\therefore u = \sqrt{\frac{17}{4}}$

$= \frac{\sqrt{17}}{\sqrt{4}} = \frac{\sqrt{17}}{2}$ is irrational.

(v) $v^2 = 3$

$\therefore v = \sqrt{3}$ is irrational.

(vi) $w^3 = 27$

$w = \sqrt[3]{27} = 3$ is rational.

(vii) $t^2 = 0.4 \therefore t = \sqrt{0.4} = \sqrt{\frac{4}{10}} = \frac{2}{\sqrt{10}}$ is irrational.

Question-11

Give an example to show that the product of a rational number and an irrational number may be a rational number.

Solution:

A rational number 0 multiplied by an irrational number gives the rational number 0.

Question-12

State with reason which of the following are surds and which are not.

(i) $\sqrt{5} \times$

(ii) $\sqrt{8} \times$

(iii) $\sqrt{27} \times \sqrt{3}$

(iv) $\sqrt{16} \times \sqrt{4}$

(v) $5\sqrt{8} \times 2\sqrt{6}$

(vi) $\sqrt{125} \times \sqrt{5}$

(vii) $\sqrt{100} \times \sqrt{2}$

(viii) $6\sqrt{2} \times 9\sqrt{3}$

(ix) $\sqrt{120} \times \sqrt{45}$

(x) $\sqrt{15} \times \sqrt{6}$.

Solution:

(i) $\sqrt{5} \times \sqrt{10} = \sqrt{5} \times \sqrt{5 \times 2} = \sqrt{5} \times \sqrt{5} \times \sqrt{2} = 5\sqrt{2}$ is a surd.

(ii) $\sqrt{8} \times \sqrt{6} = \sqrt{4 \times 2} \times \sqrt{3 \times 2} = 2 \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} = 4\sqrt{3}$ is a surd.

(iii) $\sqrt{27} \times \sqrt{3} = \sqrt{9 \times 3} \times \sqrt{3} = 3\sqrt{3} \times \sqrt{3} = 9$ is not a surd.

(iv) $\sqrt{16} \times \sqrt{4} = 4 \times 2 = 8$ is not a surd.

(v) $5\sqrt{8} \times 2\sqrt{6} = 5\sqrt{4 \times 2} \times 2\sqrt{3 \times 2} = 5 \times 2\sqrt{2} \times 2\sqrt{2} \times \sqrt{3} = 5 \times 2 \times 2 \times 2 \times \sqrt{3} = 40\sqrt{3}$ is a surd.

(vi) $\sqrt{125} \times \sqrt{5} = \sqrt{25 \times 5} \times \sqrt{5} = 5\sqrt{5} \times \sqrt{5} = 5 \times 5 = 25$ is not a surd.

(vii) $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$ is a surd.

(viii) $6\sqrt{2} \times 9\sqrt{3} = 54\sqrt{6}$ is a surd.

(ix) $\sqrt{120} \times \sqrt{45} = \sqrt{4 \times 30} \times \sqrt{9 \times 5} = 2\sqrt{6 \times 5} \times 3\sqrt{5} = 2 \times \sqrt{6} \times \sqrt{5} \times 3 \times \sqrt{5} = 30\sqrt{6}$ is a surd.

(x) $\sqrt{15} \times \sqrt{6} = \sqrt{5 \times 3} \times \sqrt{2 \times 3} = \sqrt{5} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3} = 3 \times \sqrt{10}$ is a surd.

Question-13

Give two examples to show that the product of two irrational numbers may be a rational number.

Solution:

Take a = $(2+\sqrt{3})$ and b = $(2-\sqrt{3})$; a and b are irrational numbers, but their product

$(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$ is a rational number.

Take c = $\sqrt{3}$ and d = $-\sqrt{3}$; c and d are irrational numbers, but their product = -3,
is a rational number.

Question-14

Find the value of $\sqrt{5}$ correct to two places of $\sqrt{5}$ decimal.

Solution:

We know that $2^2 = 4 < 5 < 9 = 3^2$
Taking positive square roots we get
$2 < \sqrt{5} < 3$.

Next, $(2.2)^2 = 4.84 < 5 < 5.29 = (2.3)^2$
Taking positive square roots, we have
$2.2 < \sqrt{5} < 2.3$

Again, $(2.23)^2 = 4.9729 < 5 < 5.0176 = (2.24)^2$
Taking positive square roots, we obtain
$2.23 < \sqrt{5} < 2.24$
Hence the required approximation is 2.24 as $(2.24)^2$ is nearest to 5 than $(2.23)^2$.

Question-15

Prove that $\sqrt{3} - \sqrt{2}$ is irrational.

Solution:

Let $\sqrt{3} - \sqrt{2}$ be a rational number, say r
Then $\sqrt{3} - \sqrt{2} = r$
On squaring both sides we have

$$(\sqrt{3} - \sqrt{2})^2 = r^2$$
$$3 - 2\sqrt{6} + 2 = r^2$$
$$5 - 2\sqrt{6} = r^2$$
$$-2\sqrt{6} = r^2 - 5$$
$$\sqrt{6} = -(r^2 - 5)/2$$

Now $-(r^2 - 5)/2$ is a rational number and $\sqrt{6}$ is an irrational number.
Since a rational number cannot be equal to an irrational number. Our assumption that
$\sqrt{3} - \sqrt{2}$ is rational is wrong.

Question-16

Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Solution:

Let $\sqrt{3} + \sqrt{5}$ be a rational number, say r

Then $\sqrt{3} + \sqrt{5} = r$

On squaring both sides,

$(\sqrt{3} + \sqrt{5})^2 = r^2$

$3 + 2\sqrt{15} + 5 = r^2$

$8 + 2\sqrt{15} = r^2$

$2\sqrt{15} = r^2 - 8$

$\sqrt{15} = (r^2 - 8)/2$

Now $(r^2 - 8)/2$ is a rational number and $\sqrt{15}$ is an irrational number.

Since a rational number cannot be equal to an irrational number. Our assumption that

$\sqrt{3} + \sqrt{5}$ is rational is wrong.

Question-17

Examine, whether the following numbers are rational or irrational:

(i) $(\sqrt{2} + 2)^2$

(ii) $(2 - \sqrt{2}) \times (2 + \sqrt{2})$

(iii) $(\sqrt{2} + \sqrt{3})^2$

(iv) $\frac{6}{3\sqrt{2}}$

Solution:

(i) $(\sqrt{2} + 2)^2 = (\sqrt{2})^2 + 2\sqrt{2}x2 + (2)^2 = 2 + 4\sqrt{2} + 4 = 6 + 4\sqrt{2}$.

\ It is an irrational number.

(ii) $(2 - \sqrt{2}) \times (2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2 = 4 - 2 = 2$.

\ It is a rational number.

(iii) $(\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2\sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

∴ It is an irrational number.

(iv) $\frac{6}{3\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

\ It is an irrational number.

Question-18

Prove that

(a) $2 + \sqrt{3}$ is not a rational number and

(b) $\sqrt[3]{7}$ is not a rational number.

Solution:

(a) If possible, let $2 + \sqrt{3} = a$, where a is rational.

Then, $(2 + \sqrt{3})^2 = a^2$

$7 + 4\sqrt{3} = a^2$

$\sqrt{3} = \frac{a^2 - 7}{4}$ --------(i)

Now, a is rational $\Rightarrow \frac{a^2 - 7}{4}$ is rational.

$\sqrt{3}$ is rational [from (i)]

This is a contradiction.

Hence, $2 + \sqrt{3}$ is not a rational number.

(b) If possible, let $\sqrt[3]{7} = \frac{p}{q}$, where p and q are integers, having no common factors and $q \neq 0$.

Then, $(\sqrt[3]{7})^3 = (\frac{p}{q})^3$

$\Rightarrow 7q^3 = p^3$ ------(i)

$\Rightarrow p^3$ is a multiple of 7

$\Rightarrow$ p is multiple of 7.

Let p = 7m, where m is an integer.

Then, $p^3 = 343\ m^3$ ------(ii)

$\Rightarrow 7q^3 = 343\ m^3$ [from (i) and (ii)]

$\Rightarrow q^3 = 49\ m^3 \Rightarrow q^3$ is a multiple of 7.

$\Rightarrow$ q is a multiple of 7.

Thus, p and q are both multiples of 7, or 7 is a factor of p and q.

This contradicts our assumption that p and q have no common factors.

Hence $\sqrt[3]{7}$ is not a rational number.

Question-19

Examine whether the following numbers are rational or irrational:

(i) $(3 + \sqrt{2})^2$

(ii) $(3 - \sqrt{3})(3 + \sqrt{3})$

(iii) $\frac{6}{2\sqrt{3}}$

Solution:

(i) $(3+\sqrt{2})^2 = 9 + 2 + 6\sqrt{2} = 11 + 6\sqrt{2}$, which is irrational.

(ii) $(3-\sqrt{3})(3+\sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$, which is rational.

(iii) $\frac{6}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{6} = \sqrt{3}$, which is irrational.

Question-20

Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Solution:

Irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2} \times \sqrt{3}}$, i.e $\sqrt{\sqrt{6}} = 6^{(1/4)}$

Irrational numbers lying between $\sqrt{2}$ and $6^{(1/4)}$ is $\sqrt{\sqrt{2} \times 6^{\frac{1}{4}}} = 2^{(1/4)} \times 6^{(1/8)}$

Hence two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$ are $6^{(1/4)}$ and $2^{(1/4)} \times 6^{(1/8)}$.

Question-21

Express $\frac{7}{64}$ as a decimal fraction.

Solution:

```
      0.109375
   ___________
64 | 7.000
     64
     ---
     600
     576
     ---
     240
     192
     ---
     480
     448
     ---
     320
     320
     ---
     0
```

Therefore $\frac{7}{64} = 0.109375$

Question-22

Express $\frac{12}{125}$ as a decimal fraction.

Solution:

```
        0.096
125)12.000
    1125
      750
      750
        0
```

Therefore $\frac{12}{125} = 0.096$.

Question-23

Express $\frac{451}{13}$ as a decimal fraction.

Solution:

```
       34.692307
13 | 451·0000000
     39
      61
      52
       90
       78
       120
        117
          30
          26
           40
            39
             100
              91
               9
```

Therefore **34.692307**